Local-field effect in atom optics of two-component Bose-Einstein condensates

K.V.Krutitsky *, K.-P.Marzlin †and J.Audretsch ‡
Fakultät für Physik, Universität Konstanz, Fach M 674,
D-78457 Konstanz, Germany

Abstract

Starting from the first principles of nonrelativistic QED we have developed the quantum theory of the interaction of a two-component ultracold atomic ensemble with the electromagnetic field of vacuum and laser photons. The main attention has been paid to the consistent consideration of dynamical dipole-dipole interactions in the radiation field. Taking into account local-field effects we have derived the system of Maxwell-Bloch equations. Optical properties of the two-component Bose gas are investigated. It is shown that the refractive index of the gas is given by the Maxwell-Garnett formula. All equations which are used up to now for the description of the behavior of an ultracold atomic ensemble in a radiation field can be obtained from our general system of equations in the low-density limit. Raman-Nath diffraction of the two-component atomic beam is investigated on the basis of our general system of equations.

*Permanent address: Ulyanovsk Branch of Moscow Institute of Radio Engineering and Electronics of Russian Academy of Sciences, P.B.9868, 48, Goncharov Str., Ulyanovsk 432011, Russia; e-mail: kostya@spock.physik.uni-konstanz.de, ufire@mv.ru

†e-mail: Peter.Marzlin@uni-konstanz.de

‡e-mail: Juergen.Audretsch@uni-konstanz.de

1 Introduction

In recent years a great attention has been paid to the investigation of two-component Bose-Einstein condensates (BEC). A two-component BEC can consist of spatially separated identical atoms, or it can be a binary mixture of different alkali atoms, for instance, ^{87}Rb–^{23}Na, or different isotops like ^{87}Rb–^{85}Rb, or different hyperfine states of the same alkali atoms. A number of phenomena in two-component BECs, which are not possible in single-component BECs, has been theoretically predicted and some of them have been observed in experiments. It has been shown that the BEC in a double-well potential can oscillate between the wells by quantum coherent atomic tunneling [1, 2]. Oscillations of this kind can take place also in a two-component BEC, which consists of the same atoms in different internal states [3]. Due to the nonlinearity arising from atom-atom interactions, the oscillations are expected to be supressed when the population difference of components exceeds a critical value in a process known as macroscopic quantum self-trapping (MQST) [2]. However, in the process of collisions between the condensate and noncondensate atoms MQST decays away [4]. The dynamics of spatial separation of two-component BEC has been studied in papers [5, 6].

In the present paper we shall investigate optical properties of two-component BECs interacting with off-resonant laser radiation and develop mathematical formalism for nonlinear atom optics with two-component condensates. Nonlinear atom optics with single-component condensates is a rather well studied subject. In papers [7, 8, 9, 10, 11] different mathematical formalisms for the description of nonlinear phenomena in atom optics of single-condensates has been proposed. Optical properties of the single-condensates subject to the influence of off-resonant laser radiation have been investigated in papers [9, 10, 11, 12]. However, to our knowledge, nothing has been yet done in this direction for multicomponent condensates. Following the ideas, presented in our previous papers [10, 11], we shall derive the system of Maxwell-Bloch equations for nonlinear atom optics of two-component BECs. As an application of our general theory we shall consider a diffraction of two-component atomic beam from a standing light wave and discuss the specific features of this phenomenon, which does not take place in the analogous single-component process.

2 The Hamiltonian for the two-component condensate interacting with photons

We consider a system of ultracold atoms which is a mixture of two species of two-level atoms with masses m_1, m_2, transition frequencies ω_1, ω_2, and matrix elements of the transition dipoles moments d_1, d_2. We shall describe such a system in terms of matter field operators. Let $|g_j\rangle$ and $|e_j\rangle$, $j = 1, 2$ are the vectors of the ground and excited states of the quantized atomic fields. Then the corresponding annihilation operators of the atoms in these internal states are $\hat{\psi}_{gj}$ and $\hat{\psi}_{ej}$. Matter field operators are assumed to satisfy to the bosonic equal time commutation relations and the operators of different components are assumed to commute.

The Hamiltonian of the second quantized atomic field interacting with the photons in the multipolar formulation of QED and in the electric dipole approximation can be written down in the following manner

$$\begin{aligned}
\hat{H} &= \hat{H}_A + \hat{H}_F + \hat{H}_{AI} + \hat{H}_{AF} \ , \qquad (1)\\
\hat{H}_A &= \sum_{j=1}^{2}\left[\sum_{s=g,e}\int d\mathbf{r}\, \hat{\psi}^{\dagger}_{sj}(\mathbf{r},t)\left(-\frac{\hbar^2\nabla^2}{2m_j}\right)\hat{\psi}_{sj}(\mathbf{r},t) + \int d\mathbf{r}\, \hat{\psi}^{\dagger}_{ej}(\mathbf{r},t)\hbar\omega_j\hat{\psi}_{ej}(\mathbf{r},t)\right] \ ,\\
\hat{H}_F &= \sum_{\mathbf{k}\lambda}\hbar\omega_k\hat{c}^{\dagger}_{\mathbf{k}\lambda}(t)\hat{c}_{\mathbf{k}\lambda}(t) \ , \ \hat{H}_{AI} = -\int d\mathbf{r}\,\hat{\mathbf{P}}(\mathbf{r},t)\mathbf{E}_{in}(\mathbf{r},t) \ ,\\
\hat{H}_{AF} &= -\int d\mathbf{r}\,\hat{\mathbf{P}}(\mathbf{r},t)\hat{\mathbf{D}}_{mic}(\mathbf{r},t) \ ,
\end{aligned}$$

where the operator of the microscopic displacement field is given by

$$\hat{\mathbf{D}}_{mic}(\mathbf{r},t) = \sum_{\mathbf{k}\lambda} i\sqrt{\frac{2\pi\hbar\omega_k}{V}}\mathbf{e}_{\lambda}\hat{c}_{\mathbf{k}\lambda}\exp\left(i\mathbf{k}\mathbf{r}\right) + H.c. \ , \qquad (2)$$

and the operator of the polarization field has the following form

$$\hat{\mathbf{P}} = \sum_{j=1}^{2}\hat{\mathbf{P}}_j = \sum_{j=1}^{2}\left(\hat{\mathbf{P}}^{+}_j + \hat{\mathbf{P}}^{-}_j\right) = \sum_{j=1}^{2}\mathbf{d}_j\left(\hat{\psi}^{\dagger}_{gj}\hat{\psi}_{ej} + \hat{\psi}^{\dagger}_{ej}\hat{\psi}_{gj}\right) \ . \qquad (3)$$

Here we assume that the incident electric field $\mathbf{E}_{in}(\mathbf{r},t)$ is produced by the laser, so it can be treated as a c-number function. In the Hamiltonian (1) we neglected all types of contact interaction. This approximation is valid when the saturation parameters of atomic transitions are small enough [9, 10]. We do not include into the Hamiltonian (1) a trapping potential, because our aim is to develop a theory of nonlinear atom optical processes of unconfined atomic beams.

3 Heisenberg equations of motion for the atomic and photonic operators

The Heisenberg equations of motion for the atomic and photonic operators are easily derived by from the Hamiltonian (1) and are given by:

$$\begin{aligned}
i\hbar\frac{\partial\hat{\psi}_{gj}(\mathbf{r},t)}{\partial t} &= -\frac{\hbar^2\nabla^2}{2m_j}\hat{\psi}_{gj}(\mathbf{r},t) - \mathbf{d}_j\mathbf{E}_{in}(\mathbf{r},t)\hat{\psi}_{ej}(\mathbf{r},t) \qquad (4)\\
&-\hbar\sum_{\mathbf{k}\lambda}g^{*}_{\mathbf{k}\lambda j}\hat{c}^{\dagger}_{\mathbf{k}\lambda}(t)\exp\left(-i\mathbf{k}\mathbf{r}\right)\hat{\psi}_{ej}(\mathbf{r},t) - \hbar\hat{\psi}_{ej}(\mathbf{r},t)\sum_{\mathbf{k}\lambda}g_{\mathbf{k}\lambda j}\exp\left(i\mathbf{k}\mathbf{r}\right)\hat{c}_{\mathbf{k}\lambda}(t) \ ,
\end{aligned}$$

$$\begin{aligned}
i\hbar\frac{\partial\hat{\psi}_{ej}(\mathbf{r},t)}{\partial t} &= -\frac{\hbar^2\nabla^2}{2m_j}\hat{\psi}_{ej}(\mathbf{r},t) + \hbar\omega_j\hat{\psi}_{ej}(\mathbf{r},t) - \mathbf{d}_j\mathbf{E}_{in}(\mathbf{r},t)\hat{\psi}_{gj}(\mathbf{r},t) \qquad (5)\\
&-\hbar\sum_{\mathbf{k}\lambda}g^{*}_{\mathbf{k}\lambda j}\hat{c}^{\dagger}_{\mathbf{k}\lambda}(t)\exp\left(-i\mathbf{k}\mathbf{r}\right)\hat{\psi}_{gj}(\mathbf{r},t) - \hbar\hat{\psi}_{gj}(\mathbf{r},t)\sum_{\mathbf{k}\lambda}g_{\mathbf{k}\lambda j}\exp\left(i\mathbf{k}\mathbf{r}\right)\hat{c}_{\mathbf{k}\lambda}(t) \ ,
\end{aligned}$$

$$i\hbar\frac{\partial\hat{c}_{\mathbf{k}\lambda}(t)}{\partial t} = \hbar\omega_k\hat{c}_{\mathbf{k}\lambda}(t) - \hbar\sum_{j=1}^{2} g^*_{\mathbf{k}\lambda j}\int d\mathbf{r}e^{-i\mathbf{kr}}\left[\hat{\psi}^\dagger_{ej}(\mathbf{r},t)\hat{\psi}_{gj}(\mathbf{r},t) + \hat{\psi}^\dagger_{gj}(\mathbf{r},t)\hat{\psi}_{ej}(\mathbf{r},t)\right] , \tag{6}$$

where $\mathbf{E}^{\pm}_{in}$ are the positive and negative frequency parts of the incident classical electric field. The operator products in Eqs.(4),(5),(6) are taken in normally ordered form.

The formal solution of (6) for the photon operators is

$$\begin{aligned}\hat{c}_{\mathbf{k}\lambda}(t) = \hat{c}_{\mathbf{k}\lambda}(0)\exp\left(-i\omega_k t\right) \;&+\; i\sum_{j=1}^{2} g^*_{\mathbf{k}\lambda j}\int_0^t dt'\int d\mathbf{r}'\exp\left[i\omega_k(t'-t) - i\mathbf{kr}'\right] \\ &\times \left[\hat{\psi}^\dagger_{ej}(\mathbf{r}',t')\hat{\psi}_{gj}(\mathbf{r}',t') + \hat{\psi}^\dagger_{gj}(\mathbf{r}',t')\hat{\psi}_{ej}(\mathbf{r}',t')\right] ,\end{aligned} \tag{7}$$

where the first term $\hat{c}_{\mathbf{k}\lambda}(0)$ refers to the free-space photon field and the second one goes back to the interaction with the atoms.

To study the back reaction of the photons on matter we insert (7) in (4) and (5). By doing this procedure we eliminate photons in favor of atoms. In the rotating-wave approximation we obtain the following dynamical equations for the operators of the matter fields

$$i\hbar\frac{\partial\hat{\psi}_{gj}(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2\nabla^2}{2m_j}\hat{\psi}_{gj}(\mathbf{r},t) - \mathbf{d}\hat{\mathbf{E}}^-_{loc}(\mathbf{r},t)\hat{\psi}_{ej}(\mathbf{r},t) , \tag{8}$$

$$\begin{aligned}i\hbar\frac{\partial\hat{\psi}_{ej}(\mathbf{r},t)}{\partial t} &= -\frac{\hbar^2\nabla^2}{2m_j}\hat{\psi}_{ej}(\mathbf{r},t) + \hbar\left(\omega_j + \delta_j - i\gamma_j/2\right)\hat{\psi}_{ej}(\mathbf{r},t) \\ &\quad -\hat{\psi}_{gj}(\mathbf{r},t)\mathbf{d}\hat{\mathbf{E}}^+_{loc}(\mathbf{r},t) ,\end{aligned} \tag{9}$$

where δ_j and γ_j are the Lamb shift and the spontaneous emission rate of a single atom in free space, respectively. We have introduced the operator of the local electric field

$$\begin{aligned}\hat{\mathbf{E}}^+_{loc}(\mathbf{r},t) = \;& \mathbf{E}^+_{in}(\mathbf{r},t) + i\sum_{\mathbf{k}\lambda}\sqrt{\frac{2\pi\hbar\omega_k}{V}}\mathbf{e}_\lambda\hat{c}_{\mathbf{k}\lambda}(0)\exp\left(i\mathbf{kr} - i\omega_k t\right) \\ &+\int d\mathbf{r}'\nabla\times\nabla\times\frac{\hat{\mathbf{P}}^+\left(\mathbf{r}',t-R/c\right)}{R} ,\end{aligned} \tag{10}$$

where $\nabla\times$ refers to the point $\mathbf{r}$. The polarization operator $\hat{\mathbf{P}}$ is given by eq.(3). Note that in Eq.(10) a small volume around the observation point $\mathbf{r}$ is excluded from the integration.

Eq. (10) shows that $\hat{\mathbf{E}}^{\pm}_{loc}(\mathbf{r},t)$ is a superposition of the incident field $\mathbf{E}^{\pm}_{in}(\mathbf{r},t)$, vacuum fluctuations of the photon field, and the electric field radiated by all other atoms, which has exactly the same form as in classical optics. It is this local field which drives the inner atomic transition in Eqs.(8), (9) which can be regarded as an atom-optical analogue of the optical Bloch equations [13, 14]. They describe the dynamical evolution of second quantized matter in the field of electromagnetic radiation.

4 Lorentz-Lorenz relation and the system of Maxwell-Bloch equations in atom optics of two-component BEC

4.1 Local-field correction

The solution of Eqs. (8), (9) represents a rather complicated mathematical problem because these equations contain explicitly dipole-dipole interactions. In many particular situations such a detailed microscopic description of matter is not necessary and it is more convenient to consider optical properties of the medium on a macroscopic level. This can be done by introducing the macroscopic field $\hat{\mathbf{E}}_{mac}(\mathbf{r},t)$, which satisfyes to the macroscopic Maxwell equations for a charge-free and current-free polarization medium, instead of the local field $\hat{\mathbf{E}}_{loc}(\mathbf{r},t)$ in Eqs. (8), (9).

As in Ref. [15] we can introduce the macroscopic field by setting

$$\hat{\mathbf{E}}^{\pm}_{loc}(\mathbf{r},t) = \hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r},t) + \frac{4\pi}{3}\hat{\mathbf{P}}^{\pm}(\mathbf{r},t)\ . \tag{11}$$

This equation is often called in the literature the Lorentz-Lorenz relation. It constitutes the basis of the local-field effects in classical [16], quantum [17] and nonlinear optics (see [14, 18, 19] and references therein). In the case of a classical electromagnetic field interacting with a macroscopic dielectric medium this relation can be derived from first principles under the assumption of homogeneity and isotropy of the dielectric medium. We take it here as the definition of $\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r},t)$. It can then be shown with Eqs.(3) and (10) that this $\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r},t)$ satisfies the macroscopic Maxwell equations, which can be written down in the form of the wave equation

$$\nabla\times\nabla\times\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r},t) = -\frac{1}{c^2}\frac{\partial^2\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r},t)}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial^2\hat{\mathbf{P}}^{\pm}(\mathbf{r},t)}{\partial t^2}\ , \tag{12}$$

so it is justified to call it the quantum field operator of the macroscopic electric field. At the same time this definition allows us to interpret our results on ultracold atomic gases in analogy to the interaction of light with a macroscopic dielectric medium.

4.2 Nonlinear matter equation

We substitute (11) in (8) and (9) and pass to a reference frame rotating with frequency ω_L of the incident field, which is assumed to be monochromatic, to obtain

$$i\hbar\frac{\partial\hat{\psi}_{g1}}{\partial t} = -\frac{\hbar^2\nabla^2}{2m_1}\hat{\psi}_{g1} - \frac{\hbar}{2}\hat{\Omega}^{-}_{1}(\mathbf{r})\hat{\phi}_{e1} - \frac{4\pi}{3}d_1^2\hat{\phi}^{\dagger}_{e1}\hat{\psi}_{g1}\hat{\phi}_{e1} - \frac{4\pi}{3}\mathbf{d}_1\mathbf{d}_2\hat{\phi}^{\dagger}_{e2}\hat{\psi}_{g2}\hat{\phi}_{e1}\ , \tag{13}$$

$$\begin{aligned} i\hbar\frac{\partial\hat{\phi}_{e1}}{\partial t} &= -\frac{\hbar^2\nabla^2}{2m_1}\hat{\phi}_{e1} - \frac{\hbar}{2}\hat{\psi}_{g1}\hat{\Omega}^{+}_{1}(\mathbf{r}) - \frac{4\pi}{3}d_1^2\hat{\psi}_{g1}\hat{\psi}^{\dagger}_{g1}\hat{\phi}_{e1} \\ &\quad -\hbar\left(\Delta_1 + i\gamma_1/2\right)\hat{\phi}_{e1} - \frac{4\pi}{3}\mathbf{d}_1\mathbf{d}_2\hat{\psi}_{g1}\hat{\psi}^{\dagger}_{g2}\hat{\phi}_{e2}\ , \end{aligned} \tag{14}$$

$$i\hbar\frac{\partial\hat{\psi}_{g2}}{\partial t} = -\frac{\hbar^2\nabla^2}{2m_2}\hat{\psi}_{g2} - \frac{\hbar}{2}\hat{\Omega}_2^-(\mathbf{r})\hat{\phi}_{e2} - \frac{4\pi}{3}d_2^2\hat{\phi}_{e2}^\dagger\hat{\psi}_{g2}\hat{\phi}_{e2} - \frac{4\pi}{3}\mathbf{d}_1\mathbf{d}_2\hat{\phi}_{e1}^\dagger\hat{\psi}_{g1}\hat{\phi}_{e2} , \quad (15)$$

$$\begin{aligned} i\hbar\frac{\partial\hat{\phi}_{e2}}{\partial t} &= -\frac{\hbar^2\nabla^2}{2m_2}\hat{\phi}_{e2} - \frac{\hbar}{2}\hat{\psi}_{g2}\hat{\Omega}_2^+(\mathbf{r}) - \frac{4\pi}{3}d_2^2\hat{\psi}_{g2}\hat{\psi}_{g2}^\dagger\hat{\phi}_{e2} \\ &\quad -\hbar\left(\Delta_2 + i\gamma_2/2\right)\hat{\phi}_{e2} - \frac{4\pi}{3}\mathbf{d}_1\mathbf{d}_2\hat{\psi}_{g2}\hat{\psi}_{g1}^\dagger\hat{\phi}_{e1} , \end{aligned} \quad (16)$$

with the detunings $\Delta_j = \omega_L - \omega_j - \delta_j$, $j = 1, 2$. The position dependent Rabi frequencies $\hat{\Omega}_j^\pm(\mathbf{r}) = 2\mathbf{d}_j\hat{\mathcal{E}}_{mac}^\pm(\mathbf{r})/\hbar$ are related to the macroscopic electric field.

Because we are mainly interested in atom optical problems and want to study the coherent evolution of the center-of-mass motion of the gas, we shall neglect spontaneous emission. This is valid for situations where the absolute values of the detunings are much bigger than the spontaneous emission rates and Rabi frequencies $|\Delta_j| \gg \gamma_j$, $|\Omega_j|$. In order to do this approximation self-consistently we drop in the following the vacuum fluctuations and the spontaneous emission rates γ_j from our equations. In addition we shall replace all the operators by macroscopic functions. We may, therefore, apply *the adiabatic approximation* [8, 13, 20] to (14), (16), which gives

$$\phi_{ej}(\mathbf{r}, t) = -\frac{\Omega_j^+(\mathbf{r})\psi_{gj}(\mathbf{r}, t)}{2\Delta_j^{loc}(\mathbf{r}, t)} , \quad (17)$$

where the local detuning is given by

$$\Delta_j^{loc}(\mathbf{r}, t) = \Delta_j\left\{1 - \frac{4\pi}{3}\left[\alpha_1\left|\psi_{g1}(\mathbf{r}, t)\right|^2 + \alpha_2\left|\psi_{g2}(\mathbf{r}, t)\right|^2\right]\right\} . \quad (18)$$

Here $\alpha_j = -d_j^2/\hbar\Delta_j$ is the atomic polarizability for j-th component.

Then substituting (17) in (13), (15), which eliminates the excited states, we obtain as the result a system of nonlinear equations for the ground state matter fields $\psi_{gj}(\mathbf{r}, t)$

$$i\hbar\frac{\partial\psi_{gj}(\mathbf{r}, t)}{\partial t} = \left\{-\frac{\hbar^2\nabla^2}{2m_j} + \frac{\hbar\Delta_j\left|\Omega_j^+(\mathbf{r})\right|^2}{4\left[\Delta_j^{loc}(\mathbf{r}, t)\right]^2}\right\}\psi_{gj}(\mathbf{r}, t) . \quad (19)$$

Varying the parameters in Eq.(19) we can change the nonlinear potential which is given by the second term on the r.h.s. of Eq.(19). For instance, for increasing densities and positive detunings Δ_j the local detunings grow and, correspondingly, the nonlinear term in (19) representing the coupling to the macroscopic electric field becomes smaller. On the other hand, for negative detunings the absolute values of the local detunings decrease with increasing densities and the nonlinearity becomes greater. This behavior is exactly the same as we had in a one-component medium. In a two-component medium another regime is possible which can not be reached in a one-component medium: If the signs of the detunings are different, then one can increase the densities of the components in such a manner that the values of the local detunings, and therefore of the nonlinear potentials, will remain constant.

While Eq.(19) will allow us to derive an expression for the dielectric susceptibility of a Bose gas which closely resembles that of a classical gas we have to remark that it is only valid for low enough values of parameters $\varepsilon_j = \alpha_j |\psi_{gj}|^2$. The reason is that the adiabatic approximation (17) represents the first-order term in an expansion in $1/\Delta_j$ [13]. Therefore one also should expand Eq. (19) to first order in this parameter. This procedure leads to a pair of coupled Gross-Pitaevskii equations

$$i\hbar\frac{\partial\psi_{gj}}{\partial t} = \left\{-\frac{\hbar^2\nabla^2}{2m_j} + \frac{\hbar}{4\Delta_j}\left|\Omega_j^+\right|^2\left[1 + \frac{8\pi}{3}\left(\alpha_1 |\psi_{g1}|^2 + \alpha_2 |\psi_{g2}|^2\right)\right]\right\}\psi_{gj} \ . \tag{20}$$

Equations of this type have been used, for instance, in papers [6, 21].

4.3 Optical properties of the two-component ultracold gas

Making use of the adiabatic solutions (17) we obtain the following expression for the medium polarization

$$\mathbf{P}^+(\mathbf{r},t) = \chi(\mathbf{r},t)\mathbf{E}^+_{mac}(\mathbf{r},t) \ , \tag{21}$$

where dielectric susceptibility is given by

$$\chi(\mathbf{r},t) = \frac{\sum_{j=1}^2 \alpha_j |\psi_{gj}(\mathbf{r},t)|^2}{1 - \frac{4\pi}{3}\sum_{j=1}^2 \alpha_j |\psi_{gj}(\mathbf{r},t)|^2} \ . \tag{22}$$

Dielectric susceptibility is a rather important parameter, because it describes the propagation of the laser radiation inside a medium. In most of the practical situations the electromagnetic processes are much faster than the center-of-mass motion of the atoms. Therefore, χ can be considered as a time-independent quantity. Let us assume in addition that the spatial variations of the atomic density are not very large, such that $\nabla\chi \to 0$. Then $\operatorname{div}\mathbf{E}^\pm_{mac} \approx 0$, and we have the following Helmholtz equation for the macroscopic electric field

$$\nabla^2\mathcal{E}^\pm_{mac} + k_L^2 n^2 \mathcal{E}^\pm_{mac} = 0 \ , \tag{23}$$

with the refractive index n given by the Maxwell-Garnett formula

$$n^2 = 1 + 4\pi\chi = \frac{1 + \frac{8\pi}{3}\sum_{j=1}^2 \alpha_j |\psi_{gj}|^2}{1 - \frac{4\pi}{3}\sum_{j=1}^2 \alpha_j |\psi_{gj}|^2} \ , \tag{24}$$

which is a two-component analogue of the Clausius-Mossotti formula.

Eqs. (19), (23), (24) can be considered as an atom optical analogue of the system of Maxwell-Bloch equations. In general they have to be solved in a self-consistent way and in usual situations solutions can be obtained only by doing numerical calculations. In the next section we shall consider one particular example of an analytical description of a nonlinear atom optical system.

5 Diffraction of a two-component ultracold atomic beam from a strong standing light wave

We consider a typical scheme for the observation of diffraction in atom optics: An incident atomic beam moves in z-direction, perpendicular to two laser waves counter propagating along the y-axis with wave vectors $+\mathbf{k}_L$ and $-\mathbf{k}_L$, respectively, and with Gaussian envelope. From the uncertainty relation it follows that in order to get a distinct diffraction pattern, the width of the atomic wave packet w_y should be sufficiently large compared to the wave lengh of the laser radiation in a medium. In this case the atoms can be described as a homogeneous medium with constant refractive index. If the spontaneous emission does not make any contribution, the effect of the atoms on the laser beam is purely dispersive and only the wavelength will be shifted. This means that in a medium we shall have a standing wave which is formed by counter propagating laser beams with the wave vectors $+n\mathbf{k}_L$ and $-n\mathbf{k}_L$, respectively. In this approximation the solution of (23) with (24) is given by

$$\left|\Omega_j^+\right|^2 = |\Omega_j|^2 \exp\left(-z^2/w_L^2\right) \cos^2 nk_L y \ . \tag{25}$$

We assume that the longitudinal kinetic energy of the atomic beam, associated with the center-of-mass motion in z direction, is large compared to the nonlinear potential in eq.(19). Then the z-component of the atomic velocity will not change much and, therefore, the motion of atoms in z direction during the whole evolution can be treated classically. Only the motion in y direction should be treated quantum mechanically. In such a situation the coordinate z plays the role of time and we can change the variable $t = z/v_{gj}$ in (19) with v_{gj} being the group velocity of the j-th component. In addition we assume that we are in the Raman-Nath regime and we can neglect the transverse kinetic energy during the interaction of the atoms with the electromagnetic field. This approximation is valid for heavy atoms or if the interaction is so strong that atoms can take up momentum without changing considerably the velocity [22]. In this case the density of atoms remains unaltered, but their phase changes. Making use of all these assumptions we can write down the solutions of eqs.(19) for $z \gg w_L$ (in the far zone) in the following form

$$\psi_{gj}(y,\infty) = \psi_{gj}(y,-\infty) \exp\left(\int_{-\infty}^{\infty} \frac{-i\left|\Omega_j^+(y,z)\right|^2}{4\Delta_j v_{gj}\left(1+V_1\rho_{g1}+V_2\rho_{g2}\right)^2} dz\right) , \tag{26}$$

where

$$V_j = -\frac{4\pi}{3}\alpha_j = \frac{4\pi}{3\hbar}\frac{d_j^2}{\Delta_j} , \tag{27}$$

and $\rho_{gj} = |\psi_{gj}|^2$ is the density of atoms in the ground state.

We represent ρ_{gj} as Gaussian wave packets with width w_y

$$\rho_{gj} = \rho_j \exp\left(-y^2/w_y^2\right) \ . \tag{28}$$

Then we substitute Eqs.(25) and (28) into Eq.(26) and take into account that the width of the atomic wave packet must be much larger than the wavelenght of the laser radiation, i.e., $w_y \gg 2\pi/nk_L$. After integration we get the following result

$$\psi_{gj}(y,\infty) = \psi_{gj}\left(y,-\infty\right)e^{-i\tau_j}\sum_{q=-\infty}^{\infty} e^{i2qnk_L y}(-i)^q J_q(\tau_j) \ , \tag{29}$$

which is represented here in the form of a Fourier series expansion. We use the notations:

$$\tau_j = 2g_j/\left(1+V_1\rho_1+V_2\rho_2\right)^2 \ , \ g_j = \frac{\Omega_j^2}{16\Delta_j}\frac{w_L}{v_{gj}}\sqrt{\pi} \ . \tag{30}$$

J_q is the q-th order Bessel function.

From the solution (29) it follows that the momentum transferred from the laser beam to the atomic beam is the same for both components and equals to $2qnk_L y$. It is determined by the wave number of the incident laser radiation k_L and the refractive index of the gas n. However, the probabilities to find the components of the beam in a momentum state $2qnk_L$ are different for different components:

$$P_{qj} = J_q^2(\tau_j) \ , \quad q = 0, \pm 1, \pm 2, ..., \tag{31}$$

with P_{0j} being the probability to find the j-th component in the same momentum state as for the incident atomic beam. The angle of diffraction α_{qj} for a particular momentum state q and for a particular component j is thereby given by

$$\tan\alpha_{qj} = \frac{2qn\hbar k_L}{m_j v_{gj}} \ . \tag{32}$$

Therefore the diffraction pattern, as it follows from Eqs.(29), (31), (32), depends on the densities of the components. Depending on the values of m_j and v_{gj} the angle α_{1q} can be either the same as α_{q2} or different. Only if $m_1 v_{g1} = m_2 v_{g2}$, i.e., when the momenta of different components associated with the group velocities are the same, $\alpha_{q1} = \alpha_{q2}$. In all other situations, for instance, if the group velocities of the components are equal to each other or if we have a monoenergetic atomic beam, $\alpha_{q1} \neq \alpha_{q2}$, and in the diffraction pattern one can observe spatially separated components.

6 Conclusion

In the present paper we have investigated the process of the interaction of a two-component BEC with the field of vacuum and laser photons. The two-component BEC is treated as a binary mixture of two-level atoms with different masses, transition frequencies and transition dipole moments. Starting from the microscopic model and making use of the multipolar formulation of QED, a general system of Maxwell-Bloch equations is derived which can be used for the description of nonlinear phenomena in atom optics. Optical properties of the two-component BEC are investigated. The refractive index is shown to satisfy the Maxwell-Garnett formula.

As a typical atom optical application, we have considered the diffraction of two-component atomic beam from a strong standing laser wave in the Raman-Nath approximation, which allows to obtain simple analytical solutions. It is shown that in most of the situations one can observe splitted components of the beam in the diffraction pattern.

The limits of validity of the results, obtained in the present paper, are essentially restricted by the adiabatic approximation, which is correct up to the first order with respect to the small parameters $1/\Delta_j$. Therefore, our results are valid for small enough $\varepsilon_j = \alpha_j \left|\psi_{gj}\right|^2$. They generalize our previous results [10, 11].

Although, we have considered here explicitly only a two-component BEC, the generalization to an arbitrary number of different atomic species is straightforward and can be done very easily.

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